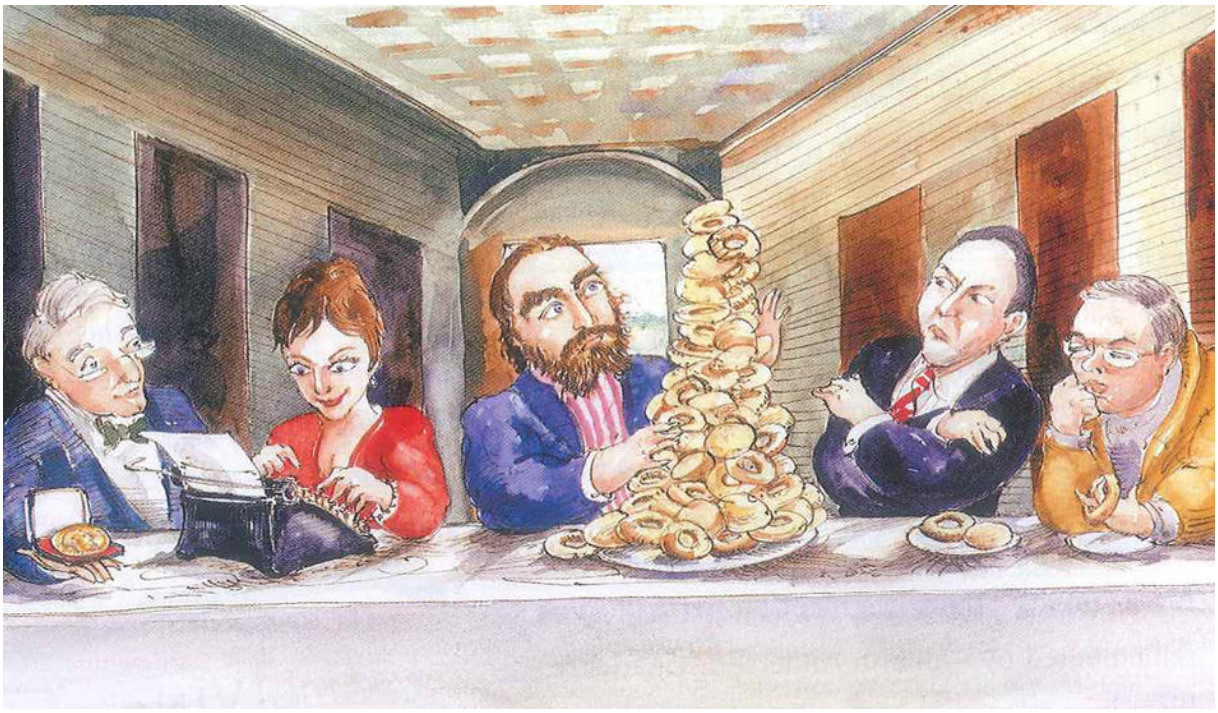
On Campus

Holes in the Argument

Who solved the Poincaré Conjecture, one of the world's most difficult math problems?

By
Paul Hond
|
Winter 2006-07



Mark Steele

Math whiz Richard Hamilton and author Sylvia Nasar probably won’t be dining together at the Faculty Club anytime soon.

Nasar, professor of business journalism at the J-School, cowrote an article in the August 28, 2006 issue of *The New Yorker* describing the battle over who solved the Poincaré Conjecture, one of the world’s most difficult math problems. The piece sparked controversy, and even the threat of a lawsuit, for its depiction of Hamilton’s close friend, Harvard mathematician Shing-Tung Yau, as having sought undeserved

credit for cracking the Poincaré. In a letter of support to Yau's attorney, Hamilton wrote: "I am very disturbed by the unfair manner in which Shing-Tung Yau has been portrayed in the *New Yorker* article. It is unfortunate that his character has been so badly misrepresented."

But Hamilton, who is Davies Professor of Mathematics at Columbia, is no mere bystander in the Poincaré affair; in fact, he did much to create it.

For a hundred years, the Poincaré Conjecture, named for the French mathematician Henri Poincaré, was a seemingly unsolvable problem that had eluded the greatest mathematical minds. To understand the Poincaré, one must grasp that, to a topologist, three-dimensional objects are no more than surfaces — that is, two-dimensional objects; a golf ball, a baseball bat, and a starfish are, in essence, the same shape, since their surfaces can all be manipulated into spheres (whereas objects with holes, like a doughnut or a pretzel, cannot). Poincaré posited that what is true for two-dimensional surfaces should also hold for three-dimensional surfaces, which do not exist inside our ordinary space, but rather inside higher-dimensional spaces. Though the analogy seemed logical, no one had been able to prove it mathematically.

In 1982 at Cornell, Hamilton published a paper on what he termed the "Ricci flow," an equation aimed at smoothing out those abstract three-dimensional shapes and producing a uniformly curved space. The process represented a huge breakthrough, and Hamilton seemed on his way to a solution. But then he hit a wall: The Ricci flow's activity, while highly promising, was found to cause all sorts of crinkles and folds upon the shapes to which it was applied. This required complicated mathematical surgeries that effectively thwarted Hamilton's efforts. The Poincaré's legendary insolubility remained intact.

Then, in 1996, Hamilton received a letter from a Russian mathematician named Grigory Perelman, a solitary figure with a scraggly beard, shoulder-length hair, and unusually long fingernails, who currently lives with his mother on the fringes of St. Petersburg. Perelman, then 30, had some ideas on how to deal with the problem of the surgeries, and expressed a wish to collaborate with Hamilton, of whom he considers himself "a disciple." (A mathematical disciple, anyway; Nasar's sketch of Hamilton as a sociable, windsurfing lady's man suggests a kind of anti-Perelman.) The response from Hamilton that Perelman awaited never came. And so Perelman decided to work alone.

Beginning in November 2002, Perelman stunned the math world by posting on the Internet, over the course of several months, a proof of the Poincaré that was as spectacularly brief as it was ambitious. If the proof withstood the scrutiny of the math establishment, Perelman would be first in line for not only the Fields Medal — the math world's equivalent, in prestige, to the Nobel Prize — but a $1 million award from the Clay Mathematics Institute. After months of expert review, including a 473-page exegesis by Columbia math chair John Morgan and MIT's Gang Tian, a consensus emerged that Perelman had indeed reached the mountaintop. (As predicted, the puristic Perelman was awarded, and refused to accept, the Fields Medal. The Clay Institute prize has yet to be announced.) In the excitement over the achievement, and with speculation swirling as to whether Perelman would accept any prizes, Richard Hamilton was given a back seat.

"Look," says Morgan, "here's an unknown guy approaching you when you've developed 20 years of work on a problem, and he's saying, 'I've got techniques that might get us to the solution; don't you want to join forces?' It's not unusual for that to happen, and it's not unusual for someone in Hamilton's position to say, 'I'm not interested.' Clearly, that was a mistake, but it's hard to know that at the time."

Yau, meanwhile, contended that Perelman's brief proof was too brief, riddled with troubling gaps and shortcuts. To fill those gaps, he enlisted two of his students from China to work on an alternate proof. According to *The New Yorker*, Yau later claimed that the students — and by extension, he himself — deserved some glory for finishing, independent of Perelman, what Richard Hamilton had begun. The lone illustration in Nasar's article depicts Yau attempting to yank the Fields Medal from Perelman's neck.

Yau quickly struck back. His attorney, Howard M. Cooper, enumerated his client's claims against *The New Yorker* in a caustic 12-page letter to the magazine, accusing Nasar and her co-writer, David Gruber '01JRN, of everything from shoddy journalism to malicious defamation. The letter, also made public on the Internet, concludes with a demand for a printed apology as well as a retraction of all erroneous passages, with the promise of further action should the magazine fail to comply. The dispute became the rage of the math blogosphere, including the popular "Not Even Wrong" blog, run by Columbia math professor Peter Woit, where the comments became so heated and partisan that Woit shut down the discussion. Even Comedy Central got into the act, with Stephen Colbert of the *Colbert Report* riffing on the Poincaré, disproving the notion that an object with a hole can't be a sphere by crushing a

doughnut in his hand and squeezing it into a ball. Such are the exponential properties of what one Poincaré-watching blogger calls “mathematainment.”

It’s a world that Sylvia Nasar knows well. She is the author of *A Beautiful Mind*, the best-selling biography of mathematician and Nobel laureate John Nash. The book was adapted into a heavily fictionalized movie starring Russell Crowe, and won the Oscar for Best Picture in 2001. Such popular attention would be anathema to a loner like Perelman, although with his eccentricities and Rasputin-like looks, he, too, would seem a shoo-in for the Hollywood treatment.

As for Richard Hamilton, the work of his own brilliant mind will, at least for the moment, have to linger in the shadows — though not entirely. “He is getting a lot of well-deserved credit,” Morgan says. “This is the second-best thing that could have happened to him.”

Read more from
Paul Hond

Guide to school abbreviations

All categories >
Read more from
Paul Hond